Special Practice Problems

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~[JEE (Mains & Advanced)]~

Topic: Limit

***Dream is not that which you see while sleeping; it is something that does not let you sleep.-A.P.J. Abdul Kalam

Objective Questions Type I [Only one correct answer]

In each of the questions below, four choices are given of which only one is correct. You have to select the correct answer which is the most appropriate.

1. If $f(x)=\frac{1}{3}\left(f(x+1)+\frac{5}{f(x+2)}\right)$ and $f(x)>0$ for all $x \in R$, then $\lim\limits_{x\to\infty} f(x)$ is

(a) 0 (b) $\sqrt{\frac{2}{5}}$

(c) $\sqrt{\frac{5}{2}}$ (d) ∞

2. The value of $\lim\limits_{x\to 0}\left(\left[\frac{100x}{\sin x}\right]+\left[\frac{99\sin x}{x}\right]\right)$, where [·] denotes the greatest integer function, is

(a) 197 (b) 198

(c) 199 (d) does not exist

3. $\lim\limits_{x\to 0}\left[\min(y^2-4y+11)\frac{\sin x}{x}\right]$ is equal to (where [.] denotes the greatest integer function)

(a) 5 (b) 6

(c) 7 (d) does not exist

4. $\lim\limits_{x\to 0}\frac{1-\cos x\cos 2x\cos 3x}{\sin^2 2x}$ is equal to

(a) 7/2 (b) 7/3

(c) 7/4 (d) 7/5

5. If $\lim\limits_{x\to\infty}(\sqrt{(x^2-x+1)}-ax-b)=0$, then for $k\geq 2$, $\lim\limits_{n\to\infty}\sec^{2n}(k!\pi b)$ is equal to

(a) $-a$ (b) a

(c) $-b$ (d) b

6. Which of the following limit is not in the indeterminant form ?

(a) $\lim\limits_{x\to a}\frac{x^3-a^3}{x-a}$ (b) $\lim\limits_{x\to 0}\sin x\operatorname{cosec} x$

(c) $\lim\limits_{x\to 0}x^x$ (d) $\lim\limits_{x\to 0}\frac{0}{x}$

7. Which of the following limit is in the indeterminant form ?

(a) $\lim\limits_{x\to\infty}\left(\frac{1}{7}\right)^x$ (b) $\lim\limits_{x\to\infty}1^x$

(c) $\lim\limits_{x\to\infty}5^x$ (d) $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

8. $\lim\limits_{n\to\infty}\frac{(1-2+3-4+5-6+\ldots-2n)}{\sqrt{(n^2+1)}+\sqrt{(4n^2-1)}}$ is equal to

(a) -1 (b) 1

(c) $-1/3$ (d) 1/3

9. $\lim\limits_{n\to\infty}\sum\limits_{x=1}^{20}\cos^{2n}(x-10)$ is equal to

(a) 0 (b) 1

(c) 19 (d) 20

10. $\lim\limits_{x\to\pi/2}\frac{\sin(x\cos x)}{\cos(x\sin x)}$ is equal to

(a) 1 (b) $\pi/2$

(c) $2/\pi$ (d) does not exist

11. $\lim\limits_{n\to\infty}{}^nC_x\left(\frac{m}{n}\right)^x\left(1-\frac{m}{n}\right)^{n-x}$ is equal to

(a) 0 (b) 1

(c) $\frac{m^x\cdot e^{-m}}{x!}$ (d) $\frac{m^x\cdot e^{m}}{x!}$

12. If $\{x\}$ and $[x]$ are the fractional part function and greatest integer functions of x respectively, then $\lim\limits_{x\to[a]}\frac{e^{\{x\}}-\{x\}-1}{\{x\}^2}$ is equal to

(a) 0 (b) 1/2

(c) $e-2$ (d) does not exist

13. $k=\lim\limits_{x\to\infty}\left(\frac{\sum\limits_{k=1}^{1000}(x+k)^m}{x^m+10^{1000}}\right)$ is $(m>101)$

(a) 10 (b) 10^2

(c) 10^3 (d) 10^4

14. If $\sum_{r=1}^{k} \cos^{-1}\beta_r = \frac{k\pi}{2}$ for any $k \ge 1$ and $A = \sum_{r=1}^{k} (\beta_r)^r$ then $\lim_{x \to A} \frac{(1+x)^{1/3} - (1-2x)^{1/4}}{x + x^2}$ is equal to

(a) 0 (b) $\frac{1}{2}$

(c) $\frac{\pi}{2}$ (d) $\frac{5}{6}$

15. If $\lim_{x \to 0} \frac{x^a \sin^b x}{\sin(x^c)}$, $a, b, c \in R \sim \{0\}$ exists and has non-zero value, then

(a) a, b, c are in AP (b) a, b, c are in GP

(c) a, b, c are in HP (d) none of these

16. Let $f(x) = \lim_{n \to \infty} n(x^{1/n} - 1)$, $x > 0$, then $f(xy)$ is equal to

(a) $f(x) + f(y)$ (b) $f(x) \cdot f(y)$

(c) $x f(y) + y f(x)$ (d) $x f(y)$

17. ABC is an isosceles triangle inscribed in a circle of radius r. If $AB = AC$ and h is altitude from A to BC, then $\lim_{h \to 0} \frac{\Delta}{P^3}$ is equal to (where Δ is the area and P is the perimeter of the triangle ABC)

(a) $\frac{1}{32r}$ (b) $\frac{1}{64r}$

(c) $\frac{1}{128r}$ (d) $\frac{1}{216r}$

18. Let $f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$. The value of $\lim_{h \to 0} \frac{f(1-h) - f(1)}{h^3 + 3h}$ is equal to

(a) 22/3 (b) 50/3

(c) 25/3 (d) 53/3

19. $\lim_{x \to 0} \left(\frac{\ln(a+x) - \ln a}{x}\right) + k \lim_{x \to e} \frac{\ln x - 1}{x - e} = 1$ then

(a) $k = e\left(1 - \frac{1}{a}\right)$

(b) $k = e(1 + a)$

(c) $k = e(2 - a)$

(d) The equality is not possible

20. $\lim_{x \to a^-} \left(\frac{|x|^3}{a} - \left[\frac{x}{a}\right]^3\right) (a > 0)$, where $[x]$ denotes the greatest integer less than or equal to x, is

(a) $a^2 - 2$ (b) $a^2 - 1$

(c) a^2 (d) $a^2 + 1$

21. If $0 < a < b$, then $\lim_{n \to \infty} (b^n + a^n)^{1/n}$ is equal to

(a) e (b) a

(c) b (d) none of these

22. The value of $\lim_{x \to 0} \frac{\sqrt{1/2(1 - \cos 2x)}}{x}$ is equal to

(a) 1 (b) −1

(c) 0 (d) none of these

23. The value of $\lim_{x \to 0} \frac{\ln(1 + \{x\})}{\{x\}}$ is (where $\{x\}$ denotes the fractional part of x)

(a) 1 (b) 0

(c) 2 (d) does not exist

24. Let $f(x) = 1/\sqrt{(18 - x^2)}$, then value of $\lim_{x \to 3} \left(\frac{f(x) - f(3)}{x - 3}\right)$ is

(a) 0 (b) −1/9

(c) −1/3 (d) 1/9

25. The value of $\lim_{x \to \infty} \left(\frac{x^2 \sin(1/x) - x}{1 - |x|}\right)$ is

(a) 0 (b) 1

(c) −1 (d) none of these

26. $\lim_{n \to \infty} \left(\frac{1}{n^3+1} + \frac{4}{n^3+1} + \frac{9}{n^3+1} + \ldots + \frac{n^2}{n^3+1}\right)$ is equal to

(a) 1 (b) 2/3

(c) 1/3 (d) 0

27. If $x > 0$ and g is bounded function, then $\lim_{n \to \infty} \frac{f(x) e^{nx} + g(x)}{e^{nx} + 1}$ is

(a) $f(x)$ (b) $g(x)$

(c) 0 (d) none of these

28. The integer n for which $\lim_{x \to 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$ is a finite non-zero number is

(a) 1 (b) 2

(c) 3 (d) 4

29. $\lim_{x \to 0} \frac{x\sqrt{y^2 - (y - x)^2}}{\left(\sqrt{(8xy - 4x^2)} + \sqrt{(8xy)^3}\right)}$ is equal to

(a) 1/4 (b) 1/2

(c) $1/2\sqrt{2}$ (d) none of these

30. $\lim_{x \to 0} \frac{\sin(\pi \cos^2 x)}{x^2}$ is equal to

(a) $-\pi$ (b) π

(c) $\pi/2$ (d) 1

31. $\lim_{x \to \pi/2} \frac{\sin x - (\sin x)^{\sin x}}{1 - \sin x + \ln \sin x}$ is equal to

(a) 4 (b) 2

(c) 1 (d) none of these

32. The value of the limit $\lim_{x \to 0} \frac{a^{\sqrt{x}} - a^{1/\sqrt{x}}}{a^{\sqrt{x}} + a^{1/\sqrt{x}}}$, $a > 1$ is

(a) 4 (b) 2

(c) −1 (d) 0

33. $\lim_{n \to \infty} \frac{n^{\alpha} \sin^2 n!}{n + 1}$, $0 < \alpha < 1$, is equal to

(a) 0 (b) 1

(c) ∞ (d) none of these

34. $\lim_{x \to 0} \frac{\sin^{-1} x - \tan^{-1} x}{x^2}$ is equal to

(a) 1/2 (b) −1/2
(c) 0 (d) ∞

35. $\lim_{n \to \infty} \{\log_{n-1}(n) \log_n(n+1) \log_{n+1}(n+2) \ldots$

$\ldots \log_{n^k - 1}(n^x)\}$ is equal to

(a) n (b) k
(c) ∞ (d) none of these

36. $\lim_{n \to \infty} \sum_{r=1}^{n} \cot^{-1}(r^2 + 3/4)$ is

(a) 0 (b) $\tan^{-1} 1$
(c) $\tan^{-1} 2$ (d) none of these

37. If $\lim_{x \to \infty} \left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x} = e^2$, then

(a) $a = 1, b = 2$ (b) $a = 2, b = 1$
(c) $a = 1, b \in R$ (d) none of these

38. $\lim_{n \to \infty} \left(\frac{1}{n} + \frac{e^{1/n}}{n} + \frac{e^{2/n}}{n} + \ldots + \frac{e^{(n-1)/n}}{n}\right)$ equals

(a) 0 (b) 1
(c) $e - 1$ (d) None of these

39. $\lim_{x \to \pi/4} \frac{2\sqrt{2} - (\cos x + \sin x)^3}{1 - \sin 2x}$ is equal to

(a) $\frac{3\sqrt{2}}{2}$ (b) $2\sqrt{2}$
(c) $\frac{4\sqrt{2}}{3}$ (d) does not exist

40. If $\lim_{x \to 0} \frac{((a-n)nx - \tan x)\sin nx}{x^2} = 0$, where n is non zero real number, then a is equal to

(a) 0 (b) $\frac{n+1}{n}$
(c) n (d) $n + \frac{1}{n}$

41. If $\lim_{x \to 0} \frac{(1 + a^3) + 8e^{1/x}}{1 + (1 - b^3)e^{1/x}} = 2$, then

(a) $a = 1,\ b = 2$ (b) $a = 1,\ b = -3^{1/3}$
(c) $a = 2,\ b = 3^{1/3}$ (d) none of these

42. If $\lim_{x \to \infty} (\sqrt{(x^4 - x^2 + 1)} - ax^2 - b) = 0$, then

(a) $a = 1, b = -2$ (b) $a = 1, b = 1$
(c) $a = 1, b = -1/2$ (d) none of these

43. If $S_n = \sum_{k=1}^{n} a_k$ and $\lim_{n \to \infty} a_n = a$, then

$\lim_{n \to \infty} \frac{S_{n+1} - S_n}{\sqrt{\sum_{k=1}^{n} k}}$ is equal to

(a) 0 (b) a
(c) $\sqrt{2}\,a$ (d) $2a$

44. The value of $\lim_{x \to 0} \frac{\cos(\sin x) - \cos x}{x^4}$ is equal to

(a) 1/5 (b) 1/6
(c) 1/4 (d) 1/2

45. Let $f(x) = \begin{cases} x^2 - 1, & \text{if } 0 < x < 2 \\ 2x + 3, & \text{if } 2 \le x < 3 \end{cases}$, the quadratic equation whose roots are $\lim_{x \to 2-0} f(x)$ and $\lim_{x \to 2+0}$ is

(a) $x^2 - 6x + 9 = 0$ (b) $x^2 - 10x + 21 = 0$
(c) $x^2 - 14x + 49 = 0$ (d) none of these

46. $\lim_{x \to 1} \frac{x \sin\{x - [x]\}}{x - 1}$, where [.] denotes the greatest integer function, is

(a) 0 (b) −1
(c) not existent (d) none of these

47. The value of $\lim_{x \to 0} [x^2 + x + \sin x]$ is (where [.] denotes the greatest integer function)

(a) does not exist (b) is equal to zero
(c) −1 (d) none of these

48. $\lim_{x \to \pi/4} (2 - \tan x)^{1/\ln(\tan x)}$ equals

(a) e (b) 1
(c) 0 (d) e^{-1}

49. $\lim_{n \to \infty} \left(\frac{n!}{(mn)^n}\right)^{1/n}$ $(m \in N)$ is equal to

(a) $1/em$ (b) m/e
(c) em (d) e/m

50. If $y = 2^{-2^{1/(1-x)}}$, then $\lim_{x \to 1+} y$ is

(a) −1 (b) 1
(c) 0 (d) 1/2

51. The graph of the function $y = f(x)$ has a unique tangent at the point $(a, 0)$ through which the graph passes. Then $\lim_{x \to a} \frac{\log_e\{1 + 6f(x)\}}{3f(x)}$ is

(a) 0 (b) 1
(c) 2 (d) none of these

52. $\lim_{h \to 0} \frac{f(2h + 2 + h^2) - f(2)}{f(h - h^2 + 1) - f(1)}$, (given that $f'(2) = 6$ and $f'(1) = 4$) is equal to

(a) does not exist (b) is equal to −3/2
(c) is equal to 3/2 (d) is equal to 3

53. If $\lim_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3} = 1$, then

(a) $a = -5/2,\ b = -1/2$ (b) $a = -3/2,\ b = -1/2$
(c) $a = -3/2,\ b = -5/2$ (d) $a = -5/2,\ b = -3/2$

54. $\lim_{x \to 0} \frac{x^n \sin^n x}{x^n - \sin^n x}$ is non zero finite, then n must be equal to

(a) 1 (b) 2
(c) 3 (d) none of these

55. $\lim_{x\to\infty}\left(\sqrt{(x^2+x)}-x\right)$ equals

(a) $\lim_{x\to 0}\frac{x+\ln(1-x)}{x^2}$ (b) $\lim_{x\to 0}\frac{e^{-x}-1+x}{x^2}$

(c) $\lim_{x\to 0}\frac{-\sqrt{x}}{\sqrt{x}+\sqrt{(x^2+2x)}}$ (d) $\lim_{x\to 0}\frac{\cos x^2-1}{x^4}$

56. If $[x]$ denotes the greatest integer less than or equal to x, then the value of $\lim_{x\to 1}(1-x+[x-1]+|1-x|)$ is

(a) 0 (b) 1
(c) -1 (d) none of these

57. $\lim_{x\to 0}\frac{1}{x}\left(\int_y^c e^{\sin^2 t}\,dt-\int_{x+y}^c e^{\sin^2 t}\,dt\right)$ is equal to (where c is a constant)

(a) $e^{\sin^2 y}$ (b) $\sin 2y e^{\sin^2 y}$
(c) 0 (d) none of these

58. If α and β be the roots of $ax^2+bx+c=0$, then $\lim_{x\to\alpha}(1+ax^2+bx+c)^{1/(x-\alpha)}$ is

(a) $a(\alpha-\beta)$ (b) $\ln|a(\alpha-\beta)|$
(c) $e^{a(\alpha-\beta)}$ (d) $e^{a|\alpha-\beta|}$

59. If $f(x)=\begin{cases}\sin x, & x\neq n\pi,\ n\in I\\ 2, & \text{otherwise}\end{cases}$ and

$g(x)=\begin{cases}x^2+1, & x\neq 0, 2\\ 4, & x=0,\\ 5, & x=2\end{cases}$ then $\lim_{x\to 0} g\{f(x)\}$ is

(a) 5 (b) 6
(c) 7 (d) 1

60. If $A_i=\frac{x-a_i}{|x-a_i|}$, $i=1,2,3,\ldots,n$ and if $a_1<a_2<a_3<\ldots<a_n$. Then $\lim_{x\to a_m}(A_1A_2\ldots A_n)$, $1\le m\le n$

(a) is equal to $(-1)^m$ (b) is equal to $(-1)^{m+1}$
(c) is equal to $(-1)^{m-1}$ (d) does not exist

61. $\lim_{x\to 1+0}\frac{\int_1^x |t-1|\,dt}{\sin(x-1)}$ is equal to

(a) 0 (b) 1
(c) -1 (d) none of these

62. $\lim_{x\to\infty}\frac{\log_e[x]}{x}$, (where [.] denotes the greatest integer function) is equal to

(a) 0 (b) 1
(c) -1 (d) nonexistent

63. If $[x]$ denotes the greatest integer $\le x$, then $\lim_{n\to\infty}\frac{1}{n^3}\{[1^2x]+[2^2x]+[3^2x]+\ldots+[n^2x]\}$ equals

(a) $x/2$ (b) $x/3$
(c) $x/6$ (d) 0

64. The value of the limit $\lim_{x\to 0}\{1^{1/\sin^2 x}+2^{1/\sin^2 x}+\ldots+n^{1/\sin^2 x}\}^{\sin^2 x}$ is

(a) ∞ (b) 0
(c) $\frac{n(n+1)}{2}$ (d) n

65. $\lim_{x\to 0}\frac{x^n-\sin x^n}{x-\sin^n x}$ is non-zero finite, then n must be equal to

(a) 1 (b) 2
(c) 3 (d) none of these

66. $\lim_{x\to\infty}\frac{\log x^n-[x]}{[x]}$, $n\in N$, ($[x]$ denotes the greatest integer less than or equal to x)

(a) has value -1 (b) has value 0
(c) has value 1 (d) does not exist

67. $\lim_{x\to 0}\frac{\sin[\cos x]}{1+[\cos x]}$ ([.] denotes the greatest integer function)

(a) equal to 1 (b) equal to 0
(c) does not exist (d) none of these

68. If α and β be the roots of $ax^2+bx+c=0$, then $\lim_{x\to\alpha}\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to

(a) 0 (b) $\frac{1}{2}(\alpha-\beta)^2$
(c) $\frac{a^2}{2}(\alpha-\beta)^2$ (d) $-\frac{a^2}{2}(\alpha-\beta)^2$

69. If x is a real number in [0, 1], then the value of $f(x)=\lim_{m\to\infty}\lim_{n\to\infty}\{1+\cos^{2m}(n!\pi x)\}$ is given by

(a) 2 or 1 according as x is rational or irrational
(b) 1 or 2 according as x is rational or irrational
(c) 1 for all x
(d) 2 or 1 for all x

70. The value of $\lim_{x\to 0}\frac{(1+x)^{1/x}-e+\frac{1}{2}ex}{x^2}$ is

(a) $\frac{11}{24}e$ (b) $-\frac{11e}{24}$
(c) $\frac{e}{24}$ (d) none of these

71. $\lim_{x\to 0}\frac{\tan([-\pi^2]x^2)-\tan([-\pi^2])x^2}{\sin^2 x}$ equals (where [.] denotes the greatest integer function)

(a) 0 (b) 1
(c) $\tan 10-10$ (d) ∞

72. Let $a=\min\{x^2+2x+3,\ x\in R\}$ and $b=\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta^2}$. The value of $\sum_{r=0}^{n} a^r\cdot b^{n-r}$ is

(a) $\frac{2^{n+1}-1}{3\cdot 2^n}$ (b) $\frac{2^{n+1}+1}{3\cdot 2^n}$

(c) $\frac{4^{n+1}-1}{3\cdot 2^n}$ (d) none of these

73. $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\left(\frac{\sin x}{x-\sin x}\right)}$ equals

(a) 1 (b) e

(c) e^{-1} (d) e^{-2}

Objective Questions Type II [One or more than one correct answer(s)]

In each of the questions below four choices of which one or more than one are correct. You have to select the correct answer(s) accordingly.

1. Let $f(x)=\lim_{n\to\infty}\frac{x^{2n}-1}{x^{2n}+1}$, then

(a) $f(x)=1$ for $|x|>1$

(b) $f(x)=-1$ for $|x|<1$

(c) $f(x)$ is not defined for any value of x

(d) $f(x)=1$ for $|x|=1$

2. $\lim_{x\to\infty}\sqrt{x}\,(\sqrt{x+1}-\sqrt{x})$ equals

(a) $\lim_{x\to 0}\frac{\ln(1+x)-x}{x^2}$

(b) $\lim_{x\to 0}\frac{1-\cos x}{x^2}$

(c) $\lim_{x\to 0}\frac{\sqrt{(1+x)}-1}{x}$

(d) $\lim_{x\to 0}\frac{\sqrt{x}}{\sqrt{x+\sqrt{(x^2+2x)}}}$

3. If $\lim_{x\to 0}(\cos x+a\sin bx)^{1/x}=e^2$, then the values of a and b are

(a) $a=1, b=2$ (b) $a=2, b=1/2$

(c) $a=2\sqrt{2}, b=\frac{1}{\sqrt{2}}$ (d) $a=4, b=2$

4. If $\lim_{x\to -a}\frac{x^7+a^7}{x+a}=7$, then the value of a is

(a) 1 (b) -1

(c) 7 (d) -7

5. If $a>0$, $b<0$, then $\lim_{x\to 0+}\frac{\sqrt{(1-\cos 2ax)}}{\sin bx}$ is equal to

(a) $\frac{a\sqrt{2}}{b}$ (b) $-\frac{a\sqrt{2}}{b}$

(c) $\frac{|a|\sqrt{2}}{|b|}$ (d) $-\frac{|a|\sqrt{2}}{|b|}$

6. If $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{\lambda/x}$, $(a, b, c, \lambda>0)$ is equal to

(a) 1, if $\lambda=1$ (b) abc, if $\lambda=1$

(c) abc, if $\lambda=3$ (d) $(abc)^{2/3}$, if $\lambda=2$

7. The value of a for which

$$\lim_{x\to 0}\frac{(e^x-1)^4}{\sin\left(\frac{x^2}{a^2}\right)\log_e\left\{1+\frac{x^2}{2}\right\}}=8, \text{ is}$$

(a) -2 (b) -1

(c) 1 (d) 2

8. If $f(x)=\left(\frac{|x|}{2+|x|}\right)^{2x}$, then

(a) $\lim_{x\to\infty}f(x)=e^{-4}$ (b) $\lim_{x\to-\infty}f(x)=e^4$

(c) $\lim_{x\to\infty}f(x)=\infty$ (d) $\lim_{x\to-\infty}f(x)=1$

9. Let $f(x)=\begin{cases}1+\frac{2x}{\lambda}, & 0\le x<1\\ \lambda x, & 1\le x<2\end{cases}$,

if $\lim_{x\to 1}f(x)$ exists, then λ is

(a) -2 (b) -1

(c) 1 (d) 2

10. Let $f(x)=\begin{cases}x^2, & x<1\\ x, & 1<x<4\\ 4-x, & x>4\end{cases}$, then

(a) $\lim_{x\to 1-}f(x)=1$ (b) $\lim_{x\to 1+}f(x)=1$

(c) $\lim_{x\to 4-}f(x)=4$ (d) $\lim_{x\to 4+}f(x)=4$

11. If $m, n\in N$, $\lim_{x\to 0}\frac{\sin x^n}{(\sin x)^m}$ is equal to

(a) 1, if $n=m$ (b) 0, if $n>m$

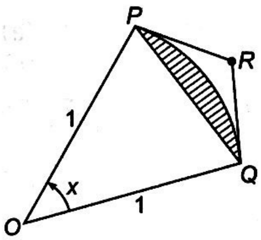
(c) ∞, if $n<m$ (d) n/m, if $n<m$

Linked-Comprehension Type

In these questions, a passage (paragraph) has been given followed by questions based on each of the passages. You have to answer the questions based on the passage given.

PASSAGE 1

A circular arc of radius 1 subtends an angle of x radians, $0 < x < \frac{\pi}{2}$ as shown in the figure. The point R is the intersection of the two tangent lines at P and Q. Let $T(x)$ be the area of triangle PQR and let $S(x)$ be the area of the shaded region.



On the basis of above information, answer the following questions :

1. The expression for $T(x)$ is

(a) $\tan\frac{x}{2}$ (b) $\tan\left(\frac{x}{2}\right)-\frac{\sin x}{2}$

(c) $\frac{\sin x}{2}$ (d) $\tan^2\frac{x}{2}-\frac{\sin x}{2}$

2. The expression for $S(x)$ is

(a) $\frac{x-\sin x}{2}$ (b) $\frac{x-\sin x}{3}$

(c) $\frac{x^2-\sin 2x}{2}$ (d) $\frac{2x-\sin^2 x}{3}$

3. The value of $\lim_{x\to 0}\frac{T(x)}{S(x)}$ is

(a) 1/2 (b) 2/3

(c) 1 (d) 3/2

4. The root of the equation $\frac{x}{2}-S(x)=T(x)\ \forall\ x\in(0,\pi)$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

5. The value of $S'\left(\frac{\pi}{2}\right)$ is

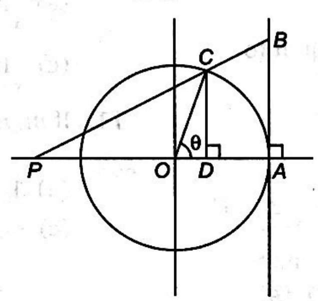
(a) -1 (b) 1/3

(c) 1/2 (d) 2/3

PASSAGE 2

A tangent line is drawn to a circle of radius unity at the point A and a segment AB is laid off whose length is equal to that of the arc AC, a straight line BC is drawn to intersect the extension of the diameter AO at the point P.

On the basis of above information, answer the following questions :



1. The area of the trapezoid $ABCD$ is

(a) $\frac{1}{2}(\theta+\sin\theta)(1-\cos\theta)$

(b) $\frac{1}{2}(1+\cos\theta)(\theta-\sin\theta)$

(c) $\frac{1}{2}(\theta+\cos\theta)(1-\sin\theta)$

(d) $\frac{1}{2}(\theta+\cos\theta)(1+\sin\theta)$

2. The length PA equals

(a) $\frac{(\theta - \sin\theta)}{\theta(1-\cos\theta)}$

(b) $\frac{\theta(1-\cos\theta)}{(\theta - \sin\theta)}$

(c) $\frac{\theta(\theta - \sin\theta)}{(1-\cos\theta)}$

(d) $\frac{(1-\cos\theta)}{\theta(\theta - \sin\theta)}$

3. The value of $\lim_{\theta\to 0^+} PA$ is

(a) 1/3 (b) 3

(c) 0 (d) none of these

4. If tangent at C intersect extended PA at Q, then the area of ΔCPQ is

(a) $\frac{1}{2}\left\{\tan\theta - \frac{\sin^2\theta(1+\theta\cot\theta)}{\theta - \sin\theta}\right\}$

(b) $\frac{1}{2}\left\{\tan\theta + \frac{\sin^2\theta(1+\theta\cot\theta)}{\theta - \sin\theta}\right\}$

(c) $\frac{1}{2}\left\{\tan\theta + \frac{\sin^2\theta(1-\theta\cot\theta)}{\theta - \sin\theta}\right\}$

(d) $\frac{1}{2}\left\{\tan\theta - \frac{\sin^2\theta(1-\theta\cot\theta)}{\theta - \sin\theta}\right\}$

5. The value of $\lim_{\theta\to 0^+} \frac{\text{area }(\Delta CPQ)}{\sin^2\theta}$ is

(a) $\frac{1}{3}$ (b) 3

(c) 0 (d) not defined

PASSAGE 3

If $\lim_{x\to 0^+} f(x) = \text{finite}$ where $f(x) = \frac{\sin x + ae^x + be^{-x} + c\ln(1+x)}{x^3}$ and a, b, c are real numbers.

On the basis of above information, answer the following questions:

1. The value of a is

(a) $-\frac{1}{2}$ (b) 0

(c) $\frac{1}{2}$ (d) 1

2. The value of b is

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$

(c) 0 (d) 1

3. The value of c is

(a) $-1/2$ (b) 1/2

(c) 0 (d) 2

4. The value of $\lim_{x\to 0^+} f(x)$ is

(a) -2 (b) -1

(c) $-\frac{1}{2}$ (d) $-\frac{1}{3}$

5. For the same values of a, b, c as obtain above, then the value of $\lim_{x\to 0^+} x^2 f(x)$ is

(a) 0 (b) 1

(c) 2 (d) 3

Answers

Objective Questions Type I [Only one correct answer]

1. (c)	**2.** (b)	**3.** (b)	**4.** (c)	**5.** (b)	**6.** (d)	**7.** (d)	**8.** (c)	**9.** (b)	**10.** (b)
11. (c)	**12.** (d)	**13.** (c)	**14.** (d)	**15.** (d)	**16.** (a)	**17.** (c)	**18.** (d)	**19.** (a)	**20.** (c)
21. (c)	**22.** (d)	**23.** (d)	**24.** (d)	**25.** (a)	**26.** (c)	**27.** (a)	**28.** (c)	**29.** (d)	**30.** (b)
31. (b)	**32.** (c)	**33.** (a)	**34.** (c)	**35.** (b)	**36.** (c)	**37.** (c)	**38.** (c)	**39.** (a)	**40.** (d)
41. (b)	**42.** (c)	**43.** (a)	**44.** (b)	**45.** (b)	**46.** (c)	**47.** (b)	**48.** (d)	**49.** (a)	**50.** (b)
51. (c)	**52.** (d)	**53.** (d)	**54.** (b)	**55.** (b)	**56.** (d)	**57.** (d)	**58.** (c)	**59.** (d)	**60.** (d)
61. (a)	**62.** (a)	**63.** (b)	**64.** (d)	**65.** (a)	**66.** (a)	**67.** (b)	**68.** (c)	**69.** (a)	**70.** (a)
71. (c)	**72.** (c)	**73.** (c)							

Objective Questions Type II [One or more than one correct answer(s)]

1. (a, b, c)	**2.** (b, c)	**3.** (a, c)	**4.** (a, b)	**5.** (a, d)
6. (c, d)	**7.** (a, d)	**8.** (a, b)	**9.** (b, d)	**10.** (a, b, c)
11. (a, b, c)				

Linked-Comprehension Type

Passage 1 **1.** (b) **2.** (a) **3.** (d) **4.** (c) **5.** (c)

Passage 2 **1.** (a) **2.** (b) **3.** (b) **4.** (c) **5.** (d)

Passage 3 **1.** (a) **2.** (b) **3.** (c) **4.** (d) **5.** (a)